Announcements

- Hw6 out due Friday as usual

Plans till spring break

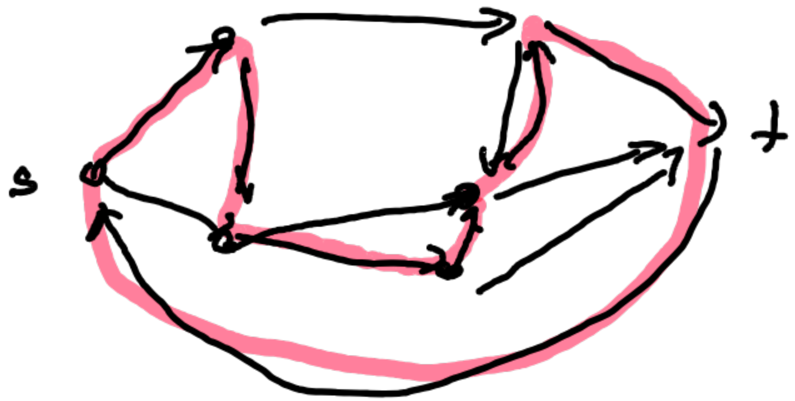
- Hw7 single question, due Tuesday March 17th
- Prelim 2: Tuesday March 24th
  - The conflicts survey is open, due on Monday, March 16th
  - Topics: stable matching, flows and applications and NP-completeness

- Mid-term survey on sections (email from me with the link)
- Reflection on mid-term course eval:

handwriting, understanding lectures, quiz fairness, grading, hw length

Sequencing problems: Traveling Salesman, (TSP) Hamiltonian Cycle, and Hamiltonian Path problems

Input directed graph $G=(V,E)$ & $c_e \geq 0$ costs



min cost cycle through all nodes

Cycle cost $\sum_{e \in C} c_e$

Problem find min cost NP-hard

not decision problem

Special case

① Hamiltonian cycle: cycle through all nodes

input G : ? does it have a Hamiltonian cycle

Clearly

Ham cycle $\leq_p$ TSP

② Hamiltonian path (s, t) path s to t going through all nodes

? does G have a Hamiltonian path

Goal: Ham. Cycle & Path NP-complete: 1. both in NP
hint: cycle or path, time $O(|V|)$

Hamiltonian Path and Cycle reduction idea

2. SAT $\leq_p$ Ham Cycle & Ham Path

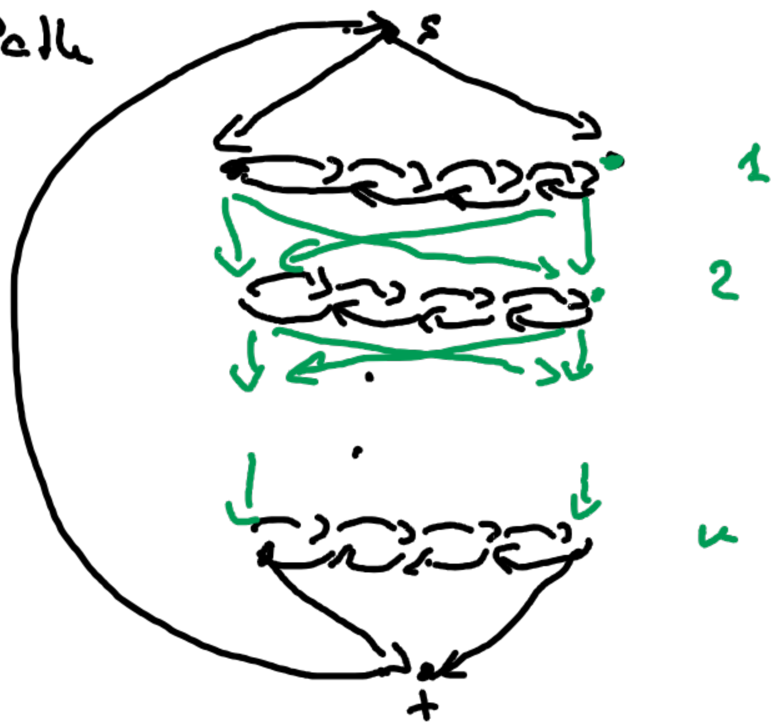
3. input to SAT ϕ

var $x_1 \dots x_n$

clauses $c_1 \dots c_m$

decision to make

$x_i \in \{T, F\}$



Join by Web **PollEv.com/evatardos772**

How many Hamiltonian Cycles are in the graph we constructed?

A: none

B: 1

C: n

D: 2^n

E: none of these

nodes s, t

v_{ij} $i = 1 \ldots n$

$j = 1 \ldots k$?

edges $(s, v_{11}), (s, v_{1k})$

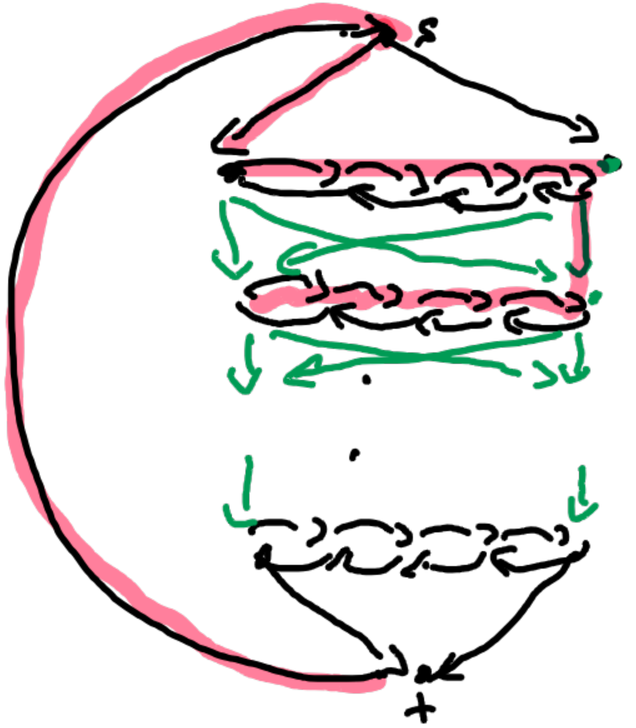
$(v_{i1}, v_{i+1\,1})$ $(v_{i1}, v_{i+1\,k})$

$(v_{ik}, v_{i+1\,1})$ $(v_{i+1\,1}, v_{i+1\,k})$

(v_{n1}, t) (v_{nk}, t)

(t, s)

(v_{ij}, v_{ij+1}) (v_{ij}, v_{ij-1})



1 $x_1 = T$

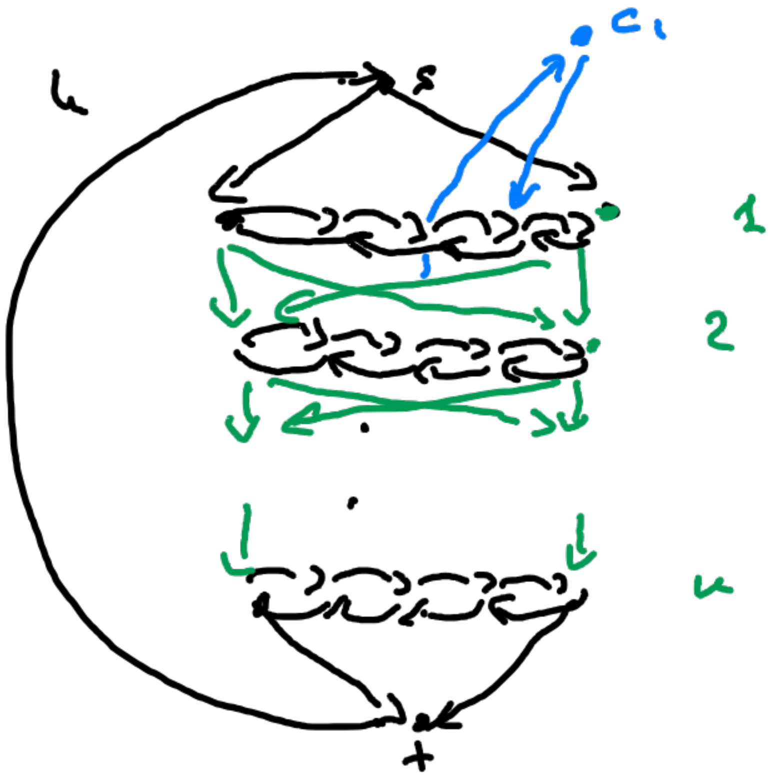
2 $x_2 = T$

n

idea: $x_i = T$ if ith line $\longrightarrow$

$x_i = F$ — " — $\longleftarrow$

Adding a simple clause to our construction



clause (x_1) clause c_1

new node c_1
edges (v_{1j}, c_1) & (c_1, v_{1j+1})

Join by Web **PollEv.com/evatardos772**

How many Hamiltonian Cycles are in the graph we constructed with the one clause node?

A: none

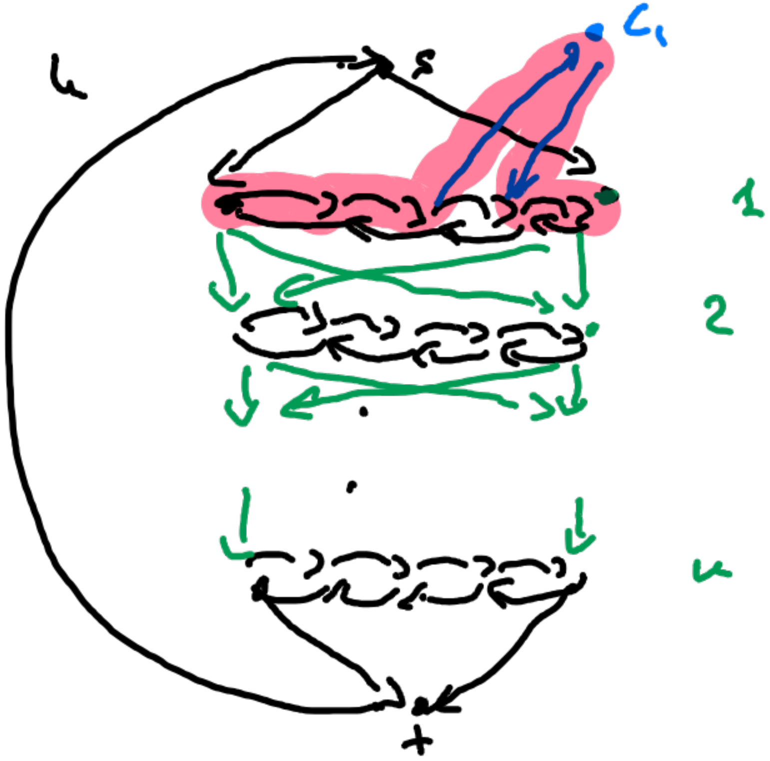
B: 1

C: n

D: 2^{n-1}

E: 2^n

F: none of these



$c_1 = (x_1)$

c_1 new node

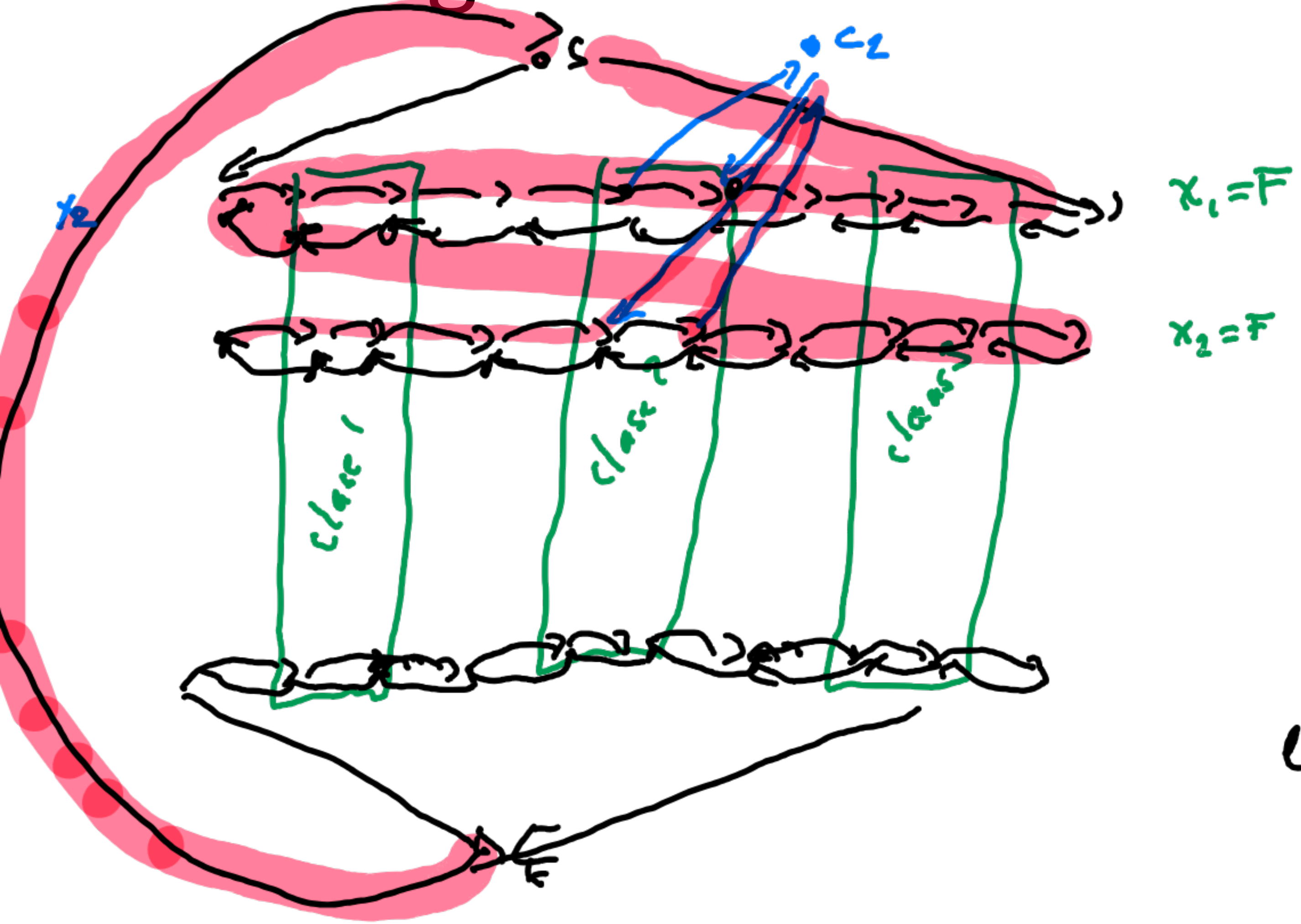
edges

$(v_{1j}\ c_1)$ (c_1, v_{1j+1})

on line ~ x_1

need to go ⟶

Adding clauses to our construction



$c_2 = (x_1 \vee \bar{x}_2)$

new nodes c_j for clause j

edges if x_i in clause

$(v_{i\,3j-2}, c_j)\ (c_j\ v_{i\,3j-1})$

if $\bar{x}_i$ in clause

$(v_{i\,3j-1}, c_j)\ (c_j, v_{i\,3j-2})$

length of path $k = 3m+1$

$m = \#$ clauses

Claim Φ satisfiable if & only if G has Ham cycle & s-t Ham Path

Proving correctness of the construction

Claim: ④ ϕ satisfyable $\Longrightarrow$ G has Ham Cycle

ϕ not satisfiable $\Longrightarrow$ G has no Ham Cycle

hard ⑤ same as G has Ham Cycle $\Longrightarrow$ ϕ satisfiable

④ ϕ satisfiable
take assignment $x_1 \dots x_n$ satisfying ϕ

$c_j = (x_1 \vee \bar{x}_2 \vee x_3) \dots$

pick one of true variables in each claus

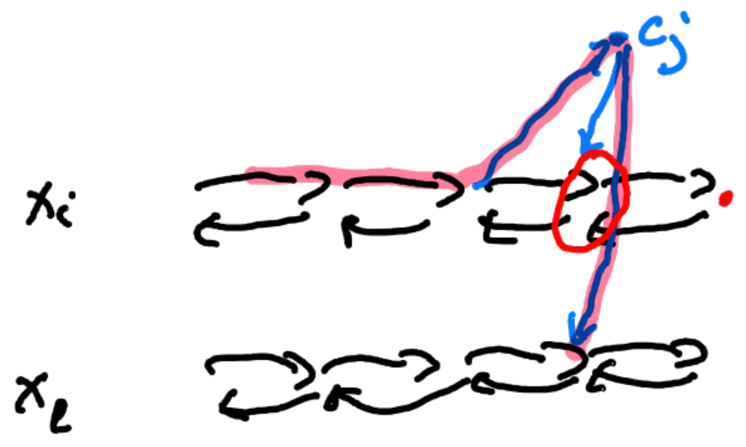
travel $\longrightarrow$ on line i if x_i true

$\longleftarrow$ — " — if x_i false

& pick up c_j between $3j-1$ & $3j-2$ nodes at the true variable

Proving correctness of the construction (cont)

(5) G has Ham. Cycle $\overset{?}{\Longrightarrow}$ ϕ is satisfiable



issue to worry about:
cycle entering c_j from line i
& then going to line $\ell \neq i$

. Claim: no cycle can do this
Proof: intended next cannot be cycle

$\Longrightarrow$ cycles correspond assignments
= goes through each line
$\longrightarrow$ or $\longleftarrow$

must be satisfying, else cannot pick up
c_j nodes ✓